Detailed SYLLABUS of the course

EXPERIMENTAL METHODS

Department of Physics, University of Trento, a.y. 2022–2023

Lecturer: **LEONARDO RICCI**

(last updated on December 14, 2022)

(12/09/2022) 1. **Introduction to the course. Fouries series (part 1/2).**

- Introduction to the course.

- Fourier series:
  - theorem (no proof);
  - behaviour of coefficients (no proof).

(13/09/2022) 2. **Fouries series (part 2/2). Fourier transform.**

- Fourier series:
  - integrability and differentiability (no proof);
  - reality;
  - Parseval's theorem.
- Transition to continuum.
- Fourier transform:
  - theorem (no proof);
  - reality;
  - Parseval's theorem.

(19/09/2022) 3. **Power spectral density, autocorrelation, Wiener-Khinchin theorem 1/2.**

- Power spectral density:
  - definition starting from the Parseval's theorem in the case of Fourier series.
- Autocorrelation function:
  - stochastic processes;
  - autocorrelation function of a stationary process via ensamble average.
- Wiener-Khinchin theorem:
  - statement and proof;
  - example of white noise.

(20/09/2022) 4. **Power spectral density, autocorrelation, Wiener-Khinchin theorem 2/2.**

- Power spectral density (from an alternative perspective):

- evaluation from Fourier transform (rather than from Fourier series).

- Autocorrelation function (from an alternative perspective):
  - evaluation via time average.

- Introduction to shot noise.

(21/09/2022) 5. **Power spectral density and signal variance. Shot noise.**

- Power spectral density and signal variance.

- Shot noise:
  - heuristic derivation;
  - derivation via Wiener-Khinchin theorem.

(27/09/2022) 6. **A macroscopic approach to fluctuations: thermodynamic fluctuations.**

- Thermodynamic fluctuations.
- Example: case of a diatomic ideal gas.

(28/09/2022) 7. **A microscopic approach to fluctuations: viscosity and Langevin equations.**

- Langevin equation for Brownian motion.
- Evolution of average velocity.
- Evolution of average position.
- Brownian motion in a medium at equilibrium: introduction.

(03/10/2022) 8. **Brownian motion.**

- Brownian motion in a medium at equilibrium.
- Brownian motion, diffusion, and Fick's law.
- A corollary: Johnson-Nyquist noise.

(04/10/2022) 9. **Spectral properties of Brownian motion.**

- Evaluation of the autocorrelation of velocity.
- Power spectral density of velocity.

- A discussion on the term *power* within “power spectral density”.

- A corollary: application to an LR circuit:
  - evaluation of the autocorrelation of current;
  - power spectral density of current.

(05/10/2022) 10. **Brownian motion of a harmonic oscillator.**

- A template problem: relation between voltage and current fluctuations in an LR circuit.
- A short discussion about the evaluation of the relaxation times.

- Brownian motion of a harmonic oscillator:
  - derivation of the PSD of the displacement from the equilibrium position;
  - fluctuation and dissipation.

- A short discussion about fluctuation, dissipation, dispersion.

(10/10/2022) 11. **Kramers-Kronig relations.**

- LTI systems.
- Causal systems.

- Kramers-Kronig relations:
  - derivation;
  - consequences on the propagation of plane waves.

(11/10/2022) 12. **Bode’s gain-phase relation.**

- A summary of Kramers-Kronig relations.
- Bode’s gain-phase relation:
  - derivation;
  - example: first-order low-pass filter.

(12/10/2022) 13. **In-class problem solving – worked examples I.**

- Dissipation and imaginary part $\chi''(\omega)$ of susceptibility $^{(\star)}$.
- Bode diagrams of low-pass filter and Bode's gain-phase relation (detailed discussion):
  - RC circuit;
  - Bode diagrams;
  - phase delay (or phase advance?) $^{(\star)}$.

$^{(\star)}$ proposed as a homework in the previous lecture.

(18/10/2022) 14. **Towards fluctuation-dissipation theorem.**

- A review of time-dependent perturbation theory:
  - Dirac's time-dependent perturbation theory (via variation of constants);
  - Fermi's golden rule.
- Quantum damped oscillator and Kramers-Kronig relations:
  - solution;
  - *en passant*: Sokhotski-Plemelj theorem.

(19/10/2022) 15. **Fluctuation-dissipation theorem 1/2.**

- The physical problem of interacting with an ideal system.
- Callen and Welton's quantum-mechanical formulation of fluctuation-dissipation theorem (1/2):
  - dissipation;
  - fluctuation (1/2).

(25/10/2022) 16. **Fluctuation-dissipation theorem 2/2.**

- Callen and Welton's quantum-mechanical formulation of fluctuation-dissipation theorem (2/2):
  - fluctuation (2/2);
  - classical limit.

(26/10/2022) 17. **In-class problem solving – worked examples II.**

- An introductory discussion of a prototypical problem: photodetection.

- Problems concerning fluctuation-dissipation theorem:
  - free particle and Brownian motion $^{(\star)}$;
  - RC circuit $^{(\clubsuit)}$;
  - LC circuit $^{(\clubsuit)}$;
  - Lorentz model and radiative damping (beginning of the discussion).

$^{(\star)}$ proposed as a homework in the previous lecture.
$^{(\clubsuit)}$ proposed as a homework.

(02/11/2022) 18. **In-class problem solving – worked examples III.**

- Problems concerning fluctuation-dissipation theorem:
  - RC circuit $^{(\star)}$;
  - LC circuit $^{(\star)}$;
  - Lorentz model and radiative damping (end of the discussion).

$^{(\star)}$ proposed as a homework in the previous lecture.

(07/11/2022) 19. **An overview on generation and detection of light.**

- Generation of light:
  - incandescence,
    * Stefan-Boltzmann law $^{(\spadesuit)}$;
  - luminescence,
    * charge-induced ⇒ electroluminescence, LEDs,
    * induced by electron collisions ⇒ cathodoluminescence, discharges,
    * induced by photons ⇒ fluorescence, phosphorescence;
  - lasers $^{(\spadesuit)}$.
- Detection of light:
  - chemical reactions,
    * retinae,
    * photographic plates;
  - heat detectors,
    * referred to as bolometers if they rely on the variation of an electric quantity, e.g. resistance;
  - photomultipliers;

- semiconductor (pn) junctions (♠),
  - $I - V$ characteristics,
  - solar cells,
  - photodiodes,
  - avalanche photodiode.

(♠) the topic was discussed in (relatively) greater detail.

(08/11/2022) 20. **Photoelectric effect. Introduction to correlation of light beams.**

- An overview on photoelectric effect:
  - Einstein equation;
  - transition probability;
  - quantum efficiency;
  - photocurrent as a function of light intensity.
- Introduction to correlation of light beams:
  - Young's double slit experiment;
  - monochromatic light;
  - thermal light;
  - collisional broadening as a Poisson point process.

(15/11/2022) 21. **Thermal light: electric field. Poisson point processes.**

- A review of last lecture:
  - photocurrent as a function of light intensity;
  - correlation of light beams.
- Thermal light in terms of electric field:
  - model of uncorrelated monochromatic emitters subject to independent, identically-distributed (i.i.d.) random phase jumps that ensue, e.g., collisions;
  - vanishing expected value (mean) of the electric field;
  - autocorrelation of the electric field;
  - Lorentzian line-shape of the power spectral density of the electric field.

- Poisson point processes:

  - defining properties,
    * $dP = \Gamma dt$,
    * independent events,
    * vanishing probability of coincidence;
  - probability distribution,
    * derivation via recursion and induction,
    * derivation via probability-generating function.

(16/11/2022) 22. **Functional descriptions of distributions and probability density functions. Central limit theorem.**

- Functional descriptions of distributions and probability density functions:
  - probability-generating function (♠);
  - moment-generating function;
  - characteristic function;
  - cumulant-generating function.

  (♠) already introduced in the previous lecture.
- Central limit theorem: derivation via characteristic function.
- Problems concerning statistics:
  - probability-generating function of Bernoulli and binomial distributions;
  - probability-generating function of the distribution of a random variable (r.v.) given by the linear combination of independent, though not necessarily identically-distributed, r.v.'s that take on integer values (♣),
    * general expression,
    * derivation of binomial distribution from Bernoulli distribution,
    * distribution of the difference between two i.i.d. r.v.;
  - derivation of the distribution of time between events in a Poisson point process (exponential distribution);
  - moments of the exponential distribution via cumulant-generating function (♣).

  (♣) proposed as a homework.

(21/11/2022) 23. **Thermal light: intensity.**

- Thermal light in terms of intensity:

- expected value (mean) of intensity;
- variance of intensity;
- distribution of intensity.

(22/11/2022) 24. **Coherence (part 1/2). Michelson interferometer.**

- Degree of first-order coherence (aka complex degree of coherence) $g^{(1)}_{1,2}(\tau)$ (aka $\gamma_{1,2}(\tau)$):
  - Young's double slit experiment revisited;
  - definition;
  - property $|g^{(1)}_{1,2}(\tau)| \leqslant 1$;
  - $g^{(1)}_{1,2}(\tau)$ as the *degree of coherence* between two obsevations points (coherent, partially coherent, incoherent);
  - relation with visibility of fringes.
- Coherence measures for the electric field:
  - mutual coherence $\Gamma_{1,2}(\tau)$;
  - self-coherence $\Gamma_{1,1}(\tau)$ and intensity $I_1 = \Gamma_{1,1}(0)$.
- Michelson interferometer and measurement of star diameter.

(23/11/2022) 25. **Coherence (part 2/2).**

- Correlation of intensity and degree of second-order coherence $g^{(2)}_{1,2}(\tau)$:
  - definition.
- Relation between $g^{(2)}_{1,2}(\tau)$ and $|g^{(1)}_{1,2}(\tau)|$ in the case of thermal light.
- Introduction to Hanbury-Brown Twiss interferometer.

(28/11/2022) Extra1. **"Office hours" 1.**

- "Office hours" session.
- Solved problems on Johnson-Nyquist noise and shot noise.
- Discussion on sampling, aliasing, and antialiasing filters.

(29/11/2022) 28. **Hanbury-Brown Twiss interferometer.**

- Hanbury-Brown Twiss interferometer:

  - exploitation of $g^{(2)}_{1,2}(\tau)$ to assess visibility;
  - structure (HPFs, delayer, mixer, filter, recorder);
  - double-balanced mixer;
  - short discussion about the effect of a finite response time by the amplifiers.
- Introduction to photon counting.

(30/11/2022) 27. **Photon counting.**

- Derivation of photon counting statistics:
  - limit of long integration times;
  - limit of short integration times and photon bunching.
- Relation with degree of coherence.

(05/12/2022) 28. **Phase-sensitive detection and lock-in amplifiers.**

- Detection of a signal in the presence of noise.
- Solution via tuned amplification.
- Solution via phase-sensitive detection (lock-in).

(12/12/2022) Extra2. **"Office hours" 2.**

- "Office hours" session.

(13/12/2022) Extra3. **"Office hours" 3.**

- "Office hours" session.
- Solved problems on experimental determination of the Boltzmann constant and on a two-state excitation/decay process.