Detailed SYLLABUS of the course

EXPERIMENTAL METHODS

Department of Physics, University of Trento, a.y. 2024–2025

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(last version of January 8, 2025)

(23/09/2024) 1. **Introduction to the course. Fourier series (part 1/2).**

- Introduction to the course.

- Fourier series:
  - theorem (no proof).

(24/09/2024) 2. **Fourier series (part 2/2).**

- Fourier series:
  - behavior of coefficients (no proof).
  - integrability and differentiability (no proof);
  - reality;
  - Parseval's theorem.
- Examples.

(25/09/2024) 3. **Wiener-Khinchin theorem (part 1/2).**

- Power spectral density:
  - definition starting from the Parseval's theorem in the case of Fourier series.
- Stochastic processes.

(30/09/2024) 4. **Wiener-Khinchin theorem (part 2/2).**

- A summary of last lecture:
  - power spectral density $S(\omega) = \lim_{T\to\infty} \frac{T}{2\pi}\langle |a_n|^2\rangle$;
  - stochastic processes.
- Autocorrelation function of a wide-sense stationary (WSS) stochastic process:
  - wide-sense stationary WSS stochastic processes;
  - autocorrelation function of a WSS stochastic process via ensamble average.
- Wiener-Khinchin theorem:
  - statement and proof.
- Alternative expressions of Wiener-Khinchin theorem:
  - discussion on the normalization term in the Fourier transform (and

Fourier transform via ν rather than ω);
  - Wiener-Khinchin theorem expressed via $\nu = \frac{\omega}{2\pi}$ and via "one-sided" PSD;
  - transition to continuum, or, from the Fourier series to the Fourier transform,
    * $\tilde{X}(\omega) = \lim_{T\to\infty} T a_n$,
    * power spectral density $S(\omega) = \lim_{T\to\infty} \frac{1}{2\pi T} \langle |\tilde{X}(\omega)|^2 \rangle$.

(01/10/2024) 5. **Mean square and variance of a stochastic process. Ergodicity. From the random telegraph to the white noise.**

- Mean square and variance of a stochastic process.
- Ergodicity:
  - general discussion.
- A WSS stationary process: *random telegraph signal*:
  - derivation of the autocorrelation function;
  - power spectral density via Wiener-Khinchin theorem.
- White noise, starting from the random telegraph signal.

(02/10/2024) 6. **Shot noise.**

- Shot noise:
  - heuristic derivation;
  - derivation of the PSD.

(07/10/2024) 7. **A microscopic approach to fluctuations: viscosity and Langevin equations.**

- Langevin equation for Brownian motion.
- Evolution of velocity.
- Evolution of position.

(08/10/2024) 8. **Brownian motion. Johnson-Nyquist noise.**

- Brownian motion of a particle in a medium at thermodynamic equilibrium.
- Johnson-Nyquist noise (aka thermal noise) as a corollary of Brownian motion.

(09/10/2024) Extra 1. **In-class problem solving & "Office hours" 1.**

- Worked examples on:
  - linear response;
  - complex integration;
  - solution of exercise 1.2.

(14/10/2024) 9. **Spectral properties of Brownian motion.**

- Evaluation of the autocorrelation of velocity.
- Power spectral density of velocity.
- A prototypical problem: Johnson-Nyquist noise in a RL circuit:
  - evaluation of the autocorrelation of current;
  - power spectral density of current;
  - relationship with the Johnson-Nyquist noise generated by the resistance.

(15/10/2024) 10. **Dampen harmonic oscillator driven by a stochastic force.**

- Derivation of the PSD of the displacement from the equilibrium position of a dampen harmonic oscillator driven by a stochastic force.
- Effect of the thermodynamic equilibrium.
- Final expression of the PSD and consequences:
  - final expression of the PSD;
  - Lorentzian-shaped PSD;
  - fluctuation and dissipation.

(16/10/2024) Extra 2. **In-class problem solving & "Office hours" 2.**

- Worked examples:
  - solution of exercise 1.1;
  - solution of exercise 1.3.

(21/10/2024) 11. **LTI systems. Kramers-Kronig relations.**

- LTI systems.
  - causal systems;
  - real systems.

- Derivation of Kramers-Kronig relations, *via Laplace formalism and L^1 integrability.*

(22/10/2024) 12. **Kramers-Kronig relations and dispersion relations.**

- A summary of Kramers-Kronig relations.
- Electromagnetic plane waves propagating in a medium and dispersion relation:
  - derivation from Maxwell's equations;
  - refraction index, electric susceptibility, dispersion, absorption.
- Dispersion relations, phase velocity, group velocity.
- Consequences of the Kramers-Kronig relations on the speed of light in a medium.
- A complementary topic: $\chi''(\omega) > 0 \Longleftrightarrow$ dissipation in the case of a real, LTI system.

(28/10/2024) 13. **Fluctuation-dissipation theorem (part 1/2).**

- A complementary topic: Susceptibility, impedance, resistance.
- The physical problem of interacting with an ideal system.
- Preliminary topics:
  - evaluation of $\langle s| \hat{Q}^2 |f\rangle$;
  - virial theorem.

(29/10/2024) 14. **Fluctuation-dissipation theorem (part 2/2).**

- Callen and Welton's quantum-mechanical formulation of fluctuation-dissipation theorem:
  - fluctuation;
  - dissipation;
  - fluctuation-dissipation theorem (FDT).
- Classical limit.

(30/11/2024) Extra 3. **In-class problem solving & "Office hours" 3.**

- Worked examples on:
  - Kramers-Kronig relations,
    - solution of exercise 2.1;
    - solution of exercise 2.5 (rainbow);
    - solution of exercise 2.2.
- "Office hours" session.

(04/11/2024) 15. **Consequences of fluctuation-dissipation theorem. Lorentz model.**

- Consequences of fluctuation-dissipation theorem:
  - FDT with regard to the time-derivative of the observable of interest;
  - FDT with regard to the driving field;
  - prototypical applications of FDT,
    - Brownian motion,
    - RC circuit.
- Lorentz model:
  - derivation;
  - link with Drude model;
  - plasma oscillations and dispersion relation.

(05/11/2024) 16. **Dissipation by radiation: black body.**

- From Lorentz model and Larmor dissipation to black-body radiation via FDT.
- Complementary topics:
  - customarily derivation of black-body radiation in terms of average energy density of the electromagnetic field;
  - average energy of a quantum harmonic oscillator in thermodynamic equilibrium with a reservoir (via partition function).
- Stefan-Boltzmann law.

(11/11/2024) 17. **Acoustic wave equation.**

- Acoustic wave equation in the one-dimensional case.

- Speed of sound and acoustic impedance.
- Generalization to the three-dimensional case.

(12/11/2024) 18. **Dissipation by radiation: sound.**

- Spherical waves generated by a pulsating sphere.
- Acoustic radiation resistance.
- Pressure fluctuations via FDT.
  Spherical waves generated by a pulsating sphere. Acoustic radiation resistance. Pressure fluctuations via FDT.

(13/11/2024) Extra 4. **In-class problem solving & "Office hours" 4.**

- Worked examples on:
  - noise in simple networks of resistances.
- "Office hours" session.

(18/11/2024) 19. **A primer in statistical methods: from probability-generating function to characteristic function.**

- Functional descriptions of distributions and probability density functions:
  - probability-generating function $G(z)$ for random variables (r.v.'s) that take on integer values;
  - moment-generating function $M(z)$;
  - cumulant-generating function $C(z)$;
  - characteristic function $F(z)$.
- Probability-generating function of an integer, linear combination of independent, though not necessarily identically-distributed, r.v.'s that take on integer values.
- Examples:
  - probability-generating function of Bernoulli and binomial distributions;
  - derivation of binomial distribution from Bernoulli distribution.
- Central limit theorem: derivation via characteristic function (part 1/2).

(19/11/2024) 20. **A primer in statistical methods: central limit theorem; Poisson point processes.**

- Central limit theorem (CLT): derivation via characteristic function (part 2/2).
- Comments on CLT:
  - the asymptotic limit is a pdf also in the case of a starting (discrete) distribution;
  - asymptotic pdf of a sum of independent, identically-distributed (i.i.d.) r.v.'s and physical interpretation of the result.
- Examples:
  - a binomial distribution asymptotically tends to a Gaussian pdf;
  - election polls and uncertainty thereof.
- Poisson point processes:
  - defining properties,
    * independent events,
    * vanishing probability of coincidence,
    * $dP = \Gamma dt$.
  - probability distribution.

(25/11/2024) 21. **An overview on generation and detection of light and on photoelectric effect. Quasi-monochromatic light sources.**

- An overview on generation and detection of light:
  - generation of light,
    * incandescence (Stefan-Boltzmann law),
    * luminescence, induced by ...
      · charges (electroluminescence, LEDs),
      · electron collisions (cathodoluminescence, discharges),
      · photons (fluorescence, phosphorescence),
    * lasers;
  - detection of light:
    * chemical reactions (retinae, photographic plates),
    * heat detectors (referred to as bolometers if they rely on the variation of an electric quantity, e.g. resistance),
    * photomultipliers,
    * semiconductor (pn) junctions,
      · $I - V$ characteristics,
      · solar cells,
      · photodiodes,
      · avalanche photodiode.

- An overview on photoelectric effect:
  - Einstein equation;
  - transition probability;
  - quantum efficiency;
  - photocurrent as a function of light intensity.

- Quasi-monochromatic light sources:
  - electric field.

(26/11/2024) 22. **Quasi-monochromatic and thermal (extended) light sources.**

- Quasi-monochromatic light sources (continuation):
  - autocorrelation of electric field and its complex amplitude;
  - light intensity;
  - average intensity of a stationary source.
- Thermal light sources.
- Statistical properties of electric field of stationary, thermal light:
  - vanishing average complex amplitude;
  - autocorrelation of complex amplitude.
- Statistical properties of intensity of stationary, thermal light:
  - average intensity;
  - autocorrelation of intensity.

(27/11/2024) Extra 5. **In-class problem solving & "Office hours" 5.**

- Worked examples on:
  - probability and statistics,
    - solution of exercises 4.1, 4.2, 4.3, combinatorial problems on permutations and combinations,
    - solution of exercise 4.4, probability of shared birthdays in a group of n people, by using Stirling's approximation for the factorial,
    - discussion on the exercise 4.6, concerning the distribution of the waiting time in a Poisson point process.
- "Office hours" session.

(04/12/2024) 23. **Coherence (part 1/2). Young's double slit experiment. Michelson stellar interferometer.**

- Coherence measures for the electric field:
  - mutual coherence $\Gamma_{1,2}(\tau)$;
  - self-coherence $\Gamma_{1,1}(\tau)$ and intensity $I_1 = \Gamma_{1,1}(0)$.
- Correlation of electric field and degree of first-order coherence $g^{(1)}_{1,2}(\tau)$.
- Young's double-slit experiment.
- Michelson interferometer and measurement of star diameter.
- A mention of Hanbury Brown and Twiss stellar interferometer.

(05/12/2024) 24. **Coherence (part 2/2). Photon counting.**

- Introduction: generation of photoelectrons.
- Photon counting distribution:
  - solution via pgf for a specific "intensity trajectory";
  - ensemble average on "intensity trajectories";
  - population mean and variance of $n(t)$;
  - relation with… (see next line).
- Correlation of intensity and degree of second-order coherence $g^{(2)}_{1,2}(\tau)$.
- Photon counting statistics (in terms of population mean and variance of $n(t)$) in the case of thermal light:
  - limit of long integration times;
  - limit of short integration times and photon bunching.
- A model of collisional broadening$^{(\star)}$:
  - autocorrelation of complex amplitude for a single emitter;
  - autocorrelation of complex amplitude and intensity of a thermal source.

$^{(\star)}$ discussion completed in lecture # 28.

(09/12/2024) 25. **Amplifiers (part 1/2).**

- Operational amplifiers as a prototypical implementation of two-port networks.
- Basics of operational amplifiers (dynamical behavior in the frequency domain):

  - $V_{\text{out}} = A(V_+ - V_-)$, with $A = \frac{A_0}{1-i\nu/\nu_0}$;
  - general amplifier configuration.
- Noise and operational amplifiers (1/2):
  - external noise sources in the case of the general amplifier configuration;
  - internal voltage and current noise sources.

(10/12/2024) 26. **Amplifiers (part 2/2).**

- Noise and operational amplifiers (2/2):
  - expression in the case of the general amplifier configuration.
- Examples:
  - noise generated by buffers based on different operational amplifiers (OP07 and LM411);
  - noise generated by an operational amplifier (LM411) with $G_0 = 10^3$ gain (inverting input connected to ground via 1 kΩ; feedback resistance of 1 MΩ) and non-inverting input connected to ground (left as homework exercise);
  - prevalence of shot vs. thermal noise in a resistor (left as homework exercise).

(11/12/2024) Extra 6. **In-class problem solving & "Office hours" 6.**

- Worked examples on:
  - determination of the equivalent noise bandwidth (ENBW) in the case of a n-order low-pass filter;
  - prevalence of shot vs. thermal noise in a resistor (see lecture of 10/12/2024).
- "Office hours" session.

(16/12/2024) 27. **In-class problem solving (1/2).**

- Solution of the problems of the written exam of 20/12/2023, about:
  - systems and signals;
  - noise;
  - probability and statistics.

(17/12/2024) 28. **Complementary topics. In-class problem solving (2/2).**

- Complementary topics:
  - distribution of the waiting time (for an event to occur) in a Poisson process;
  - a model of collisional broadening (see lecture of 05/12/2024);
  - thermal noise of a complex impedance (corollary: thermal noise of resistors in series and in parallel).
- Solution of the problem of the written exam of 12/01/2024, about
  - noise.

(18/12/2024) Extra 7. **In-class problem solving & "Office hours" 7.**

- Worked examples on:
  - problem 3 of the written exam of 06/02/2024 (about noise).
- "Office hours" session.

(08/01/2025) Extra 8. **In-class problem solving & "Office hours" 8.**

- Worked example on:
  - noise generated by an operational amplifier (LM411) with $G_0 = 10^3$ gain (inverting input connected to ground via 1 kΩ; feedback resistance of 1 MΩ) and non-inverting input connected to ground (see lecture of 10/12/2024).
- "Office hours" session.