Detailed SYLLABUS of the course

LABORATORY OF ADVANCED ELECTRONICS

Department of Physics, University of Trento, a.y. 2018–2019

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<u>LECTURES</u>

(17/09/2018) 1. **Introduction to the course.**

- Main topics of the course.
- Signals processing:
  - analog signal processing;
  - noise,
    - Johnson noise,
    - shot noise,
    - quantum noise;
  - digital signal processing (DSP).

(19/09/2018) 2. **Signals and Systems. Sequences and LTI Systems. Basic properties of LTI systems.**

- Signals and Systems.
  - a formal definition of signals and systems.
  - sequences as discrete–time signals.

- Sequences:
  - definition;
  - graphical representation;
  - remarkable sequences: impulse–sequence $\delta[n]$, step–sequence $u[n]$;

  - delayed sequences, relation between $\delta[n]$ and $u[n]$;
  - representation of a generic sequence by means of delayed impulse–sequences;

  - periodic sequences;
  - energy of a sequence.
- Linear, time–invariant (LTI) systems:
  - action on a sequence, impulse–response (a.k.a. transfer function) $h[n]$, and convolution.
- Properties of an LTI system:

- reality;
- causality;
- marginal stability;
- *bound–input, bound–output* ("BIBO") stability,
  * definition,
  * necessary and sufficient condition for BIBO stability.

(24/09/2018) 3. **Difference equations. z–transform.**

- Difference equations, FIR and IIR systems:
  - general expression of a LTI system described by a difference equation (a.k.a. a linear recurrence relation);
  - non–recursive systems (FIR);
  - recursive systems (IIR);
  - examples,
    * FIR $h[n]$ in the case of $y[n] = x[n] + x[n-1]$,
    * IIR $h[n]$ in the case of $y[n] = a \cdot y[n-1] + b \cdot x[n]$.
- z–transform:
  - a short discussion on the importance of changing space, in analogy with continuous systems, to solve difference equations;
  - definition and region of convergence ("ROC");
  - remarkable examples,
    * $\delta[n]$,
    * $u[n]$, $u[-n-1]$.
- Basic properties of z–transform:
  - linearity (important: beware of intersecting ROCs!);
  - time–shift;
  - convolution theorem.
- Complementary topics: graphical representation of systems:
  - linear combinations of systems;
  - cascade systems, and invertibility of two systems (proof via z–transform).

(26/09/2018) 4. **Inversion of the z–transform.**

- Summary of residue calculus:
  - $\frac{1}{2\pi i} \oint_{\Gamma\ um\ z_o} (z - z_o)^n dz = \delta_{n,\ -1}$;

- $G(z)$ has an n^{th}–order pole in $z_o \Longrightarrow$
$\frac{1}{2\pi i}\oint_{\Gamma\ um\ z_o} G(z)dz = \frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}[G(z)(z-z_o)^n]_{z=z_o}$.

- Inversion of the z–transform:
  - proof;
  - remarkable examples,
    - $X(z) = 1$ with ROC $\mathbb{C}$,
    - (evaluation of the z–transform of $a^n u[n]$, $-a^n u[-n-1]$,)
    - $X(z) = \frac{z}{z-a}$ with ROC $|z| < |a|$,
    - $X(z) = \frac{z}{z-a}$ with ROC $|z| > |a|$;
  - exotic, remarkable examples,
    - Fibonacci sequence,
    - Poisson process and distribution (left as a homework).

(01/10/2018) 5. **Basic properties of LTI systems as seen in the z–domain. Nyquist–Shannon sampling theorem.**

- Basic properties of systems as seen in the z–domain:
  - reality,
    - reality $\Leftrightarrow \overline{X}(\overline{z}) = X(z)$,
    - existence of complex-conjugate zeroes and poles;
  - BIBO stability,
    - BIBO stability $\Leftrightarrow \Gamma_1 \subset$ ROC,
    - example;
  - causality,
    - beware: causal $\neq \overline{\text{noncausal}}$ ($k \cdot \delta[n] \dots$ is both!),
    - $x[n] = 0$ for $n < 0$ (causal) $\Leftrightarrow 0 \notin$ ROC, $\infty \subset$ ROC
      ($\Leftarrow$ Taylor series),
    - $x[n] = 0$ for $n > 0$ (noncausal) $\Leftrightarrow 0 \in$ ROC, $\infty \not\subset$ ROC
      ($\Leftarrow$ Taylor series),
    - $x[n] = 0$ for $n \neq 0$ (causual & noncausal; $x[n] = k \cdot \delta[n] \dots$) $\Leftrightarrow$
      $0 \in$ ROC, $\infty \subset$ ROC $\Leftrightarrow X(z)$ uniform on $\mathbb{C}$
      ($\Leftarrow$ Liouville theorem: a bounded holomorphic function whose ROC coincides with $\mathbb{C}$ is uniform),
    - example;

- Sampling:
  - generation of a sequence $x[n]$ starting from a continuous signal $x(t)$,
    $x[n] = x(n \cdot T)$;

- basic issue: how good a continuous signal can be reconstructed from a sequence.

- Important definitions:
  - sampling time/period;
  - sampling frequency/rate;
  - Nyquist frequency;
  - Nyquist band.
- Nyquist–Shannon sampling theorem:
  - conditions: 1a) a continuous signal $x(t)$ is L^2 and 1b) its sampled sequence $x[n]$ BIBO stable;
  - relation between the $\mathcal{F}$–transform $\tilde{X}(\omega)$ of $x(t)$ and the z–transform $X(z)$ of $x[n]$,
    $X\left(e^{-i\omega T}\right) = \frac{1}{T}\sum_{\forall k}\tilde{X}(\omega + \frac{2\pi}{T}k)$ for $\omega \in \left(-\frac{\pi}{T}, \frac{\pi}{T}\right)$ and thus, because of the periodicity, $\forall\omega$.
  - additional condition: 2) the continuous signal $x(t)$ is $\frac{\pi}{T}$–BL (band–limited);
  - relation between the $\mathcal{F}$–transform $\tilde{X}(\omega)$ of $x(t)$ and the z–transform $X(z)$ of $x[n]$,
    $\tilde{X}(\omega) = TX\left(e^{-i\omega T}\right)$ if $\omega \in \left(-\frac{\pi}{T}, \frac{\pi}{T}\right)$;
  - theorem's statement and proof.

(08/10/2018) 6. **Aliasing**

- Summary of last lecture:
  - definitions (sampling time/period, sampling frequency/rate, Nyquist frequency, Nyquist band);
  - sampling $x[n] = x(nT)$;
  - relation (Nyquist–Shannon precursor equality)
    $T \cdot X\left(e^{-i\omega T}\right) = \sum_{\forall k}\tilde{X}(\omega + \frac{2\pi}{T}k)$
    under the conditions 1a) $x(t)$ is L^2 and 1b) $x[n]$ is BIBO stable;
  - Nyquist–Shannon sampling theorem
    $T \cdot X\left(e^{-i\omega T}\right) = \tilde{X}(\omega)$
    under the condition 2) $x(t)$ is $\frac{\pi}{T}$–BL.
- Alternative formulation of Nyquist–Shannon sampling theorem:
  - definition of the *reconstruction* $x_{rec}(t)$ as $x_{rec}(t) \equiv \sum_{\forall k} x[k]\,\mathrm{sync}\left[(t-kT)\frac{\pi}{T}\right]$;
  - theorem statement: if $x(t)$ is $\frac{\pi}{T}$–BL (and L^2, and $x[n]$ BIBO stable),

$x_{rec}(t) = x(t)$.

- Aliasing in the case of a non–$\frac{\pi}{T}$–BL signal $x(t)$:
  - additional definitions,
    - $\tilde{X}_{folding}(\omega) \equiv \sum_{\forall k} \tilde{X}(\omega + \frac{2\pi}{T}k)$,
    - $W(\omega) \equiv [\theta(\omega + \frac{\pi}{T}) - \theta(\omega - \frac{\pi}{T})]$ ($\frac{\pi}{T}$–window),
    - $\tilde{X}_{alias}(\omega) \equiv \tilde{X}_{folding}(\omega) \cdot W(\omega)$,
    - $x_{alias}(t) \equiv \mathcal{F}^{-1}\left(\tilde{X}_{alias}(\omega)\right)$;
  - properties of $\tilde{X}_{alias}(\omega)$,
    - $\tilde{X}_{alias}(\omega)$ is $\frac{\pi}{T}$–BL,
    - $\sum_{\forall k} \tilde{X}_{alias}(\omega + \frac{2\pi}{T}k) = \sum_{\forall k} \tilde{X}(\omega + \frac{2\pi}{T}k)$;
  - aliasing: if $x(t)$ is not $\frac{\pi}{T}$–BL,
    $x_{rec}(t) = x_{alias}(t) \neq x(t)$,
    i.e. $x_{rec}(t)$ is not equal to $x(t)$ but it is equal to *something else* (in latin *alias*), namely $x_{alias}(t)$;
  - corollary: $x_{alias}[n] = x_{alias}(nT) = x(nT) = x[n]$.
- Remarkable examples: aliasing in the case of sinusoidal and cosinusoidal signals.

(15/10/2018) 7. **Simulation of an analog system by means of a digital one.**

- *Simulation theorem*:
  - general discussion;
  - proof assuming 1a) L^2 input signals and transfer function, 1b) a BIBO-stable simulator, 2) $\frac{\pi}{T}$–BL input signals;
  - expression of the ideal transfer function $h'[n]$ of the simulator.
- Implementation issues and solution:
  - difficulty – in the general case – of calculating $h'[n]$ (example: first–order low–pass filter);
  - reality: $h(t)$ is real $\Rightarrow h'[n]$ is real;
  - stability of the ideal transfer function $h'[n]$, to be assessed case by case, and ...
  - non–causality issue for the ideal transfer function $h'[n]$ (example: first–order low–pass filter).
- Backward interpretation of the simulation theorem:

- an approach to overcome the implementation issues: requiring the simulator's output $g'[n]$ to approximate the sampled analog output $g[n]$ rather than imposing $g'[n] = g[n]$. So,
  because it is mostly impossible to find $H'(z)$, and thus $h'[n]$, such that it exactly simulates a given $\tilde{H}(\omega)$,
  find an implementable $V(z)$, and thus $v[n]$, such that its Fourier–transform equivalent function $V(e^{-i\omega T})$ suitably approximates $\tilde{H}(\omega)$;
- (approximation of an ideal system $h'[n]$, $H'(z)$ through a real one $v[n]$, $V(z)$ via, ex. gr., minimization of the Tchebycheff error or the root–mean–square error;)
- a remarkable example: digital differentiator,
  * ideal solution $\propto (1 - \delta[n])\frac{(-1)^n}{n}$,
  * solution $\delta[n] - \delta[n-1]$,
  * solution via a non recursive (FIR) filter based on delays up to 2 periods.

(22/10/2018) 8. **Simulation via bilinear transform: part 1 of 2.**

- Bilinear transform:
  - bilinear transform statement $\omega \to \frac{2i}{T}\frac{z-1}{z+1}$, $s \to \frac{2}{T}\frac{z-1}{z+1}$
    $\Longrightarrow V(z) = \tilde{H}\left(\omega = \frac{2i}{T}\frac{z-1}{z+1}\right)$; $V(z) = \tilde{H}_{\text{Laplace}}\left(s = \frac{2}{T}\frac{z-1}{z+1}\right)$;
  - desirable frequency behaviour for $T \ll bandOfInterest^{-1}$.
- Example: design of a first–order low–pass filter via bilinear transform:
  - derivation of $V(z)$ via bilinear transform;
  - implementation via a difference equation;
  - transfer function $\tilde{V}(\omega)$ via backward interpretation of the simulation theorem;
  - desirable behaviour in the neighbourhood of the Nyquist frequency.
- A summary on Bode diagrams (and on frequency roll-off in filters).

(29/10/2018) 9. **Simulation via bilinear transform: part 2 of 2. Filter design.**

- Simulation via bilinear transform of a system characterized by a rational transfer function:[*]
  - systems characterized by rational transfer functions $\tilde{H}(\omega) = N(\omega)/D(\omega)$;
  - desirable behaviour in the neighbourhood of the Nyquist frequency

in the case degree$[N(\omega)] <$ degree$[D(\omega)]$;

- basic properties,
  - reality, $h(t)$ real $\Leftrightarrow \tilde{H}^*(\omega) = \tilde{H}(-\omega)$,
  - causality (to be imposed),
  - BIBO–stability, $h(t)$ BIBO–stable $\Leftrightarrow$ stable system with rational transfer function $\tilde{H}(s)$ ($\Leftarrow \mathrm{Re}\, s < 0 \leftrightarrow |z| < 1$);
- implementation of a simulator via a difference equation.

[*] generalization of the first–order low–pass filter example discussed in the previous lecture.

- Implementation via "direct forms" of systems characterized by a difference equation:
  - implementation via direct form I (DFI);
  - implementation via canonical direct form II (DFII), and its transposed version;
  - example,
    - first–order low–pass filter.
- Structure of a filter implementing a rational function in the z–domain:
  - effect of zeroes and poles on $\tilde{V}(\omega)$;
  - example: first–order low–pass filter;
  - example (left as a homework): design of a notch filter at $f = f_{Nyquist}/2 = f_{sampling}/4$.

(05/11/2018) 10. **Discrete–Fourier–Transform (DFT).**

- Discrete–Fourier–Transform (DFT):
  - assumption of the periodicity of $x(t)$, with period NT and calculation of the coefficients of the Fourier series;
  - assumption of the $\frac{\pi}{T}$–band–limitedness of $x(t)$;
  - (assumption of N being even);
  - derivation of DFT and relation with the Fourier series coefficients;
  - inversion, periodicity, Parseval's theorem.
- Matrix representation of DFT:
  - notation,
    - $f_n \equiv f[n]$,
    - *twiddle factor* $W_N \equiv e^{2\pi i/N}$;
  - matrix representation;

  - dependence $O(N^2)$ of the number of operations required for the DFT on the dimension N of the input string.
- Short mention on DFT of a nonperiodic function and windowing.

- Example: DFT of $\cos(2\pi f t)$ with $f = 2 \cdot \frac{1}{NT}$ in the case $N = 8$.

(12/11/2018) 11. **Fast–Fourier–Transform (FFT) algorithm.**

- Short summary of last lecture.
- *Radix–2 decimation–in–time* Fast–Fourier–Transform (FFT) algorithm:
  - short history: Gauss 1805, Cooley and Tukey 1965;
  - Danielson–Lanczos lemma (*decimation*, i.e. split);
  - *butterfly* diagram (basic computational unit);
  - element ordering via *bit swapping*.
- Dependence $O(N \log N)$ of the number of operations required for the FFT on the dimension N of the input string.

- Examples of FFT in the case $N = 8$:
  - FFT of $\cos(2\pi f t)$ with $f = 2 \cdot \frac{1}{NT}$;

  - ...proposed as a homework,
    * generic sequence $\{f_0, f_1, \ldots, f_7\}$, and equivalence with the DFT,
    * FFT of $\sin(2\pi f t)$ with $f = 2 \cdot \frac{1}{NT}$,
    * FFT of a constant,
    * FFT of $\delta[n]$,
    * FFT of $\delta[n-1]$,
    * FFT of $\delta[n-4]$.

(19/11/2018) 12. **An introduction to information theory: Shannon entropy of an ensamble.**

- The basic issue: a definition of information, i.e., how it can be measured:
  - Shannon's approach (1948) with uncertainty;
  - example of a sport match.
- Ensambles (Khinchin's finite schemes) as triples made of...:
  - random variable;
  - alphabet of symbols, each corresponding to a realization of the r.v.;

    - probability distribution of the r.v.
- Shannon entropy of an ensamble:
    - Shannon-Khinchin axioms;
    - proof of uniqueness theorem;
    - examples.
- Statement of Shannon's source coding theorem:
    - (uniquely decodable) binary symbol codes and compression;
    - expected length $\ell(C, X)$ of a symbol code C encoding an ensamble X;
    - compression issue: given an ensamble X, generate a symbol code C that...
        * is uniquely decodable, and
        * minimizes the expected length $\ell(C, X)$;
    - statement of Shannon's source coding theorem.
- A mention of Shannon's noisy channel coding theorem.

(26/11/2018) A1. **An introduction to information theory: Shannon's source coding theorem for symbol codes; compression.**
Additional lecture. Duration: 2h.

- Summary of lecture 12 (with emphasis on the concept of Shannon information content of an outcome).

- Binary symbol codes and compression issue:
    - definitions related to symbol codes,
        * (binary) symbol code, codewords, length of codewords,
        * extended code,
        * uniquely decodable symbol code and prefix code,
          a prefix code is uniquely decodable; the contrary is not true (counterexample $\{1, 101\}$),
          a prefix code is (generally) easy to decode,
        * examples;
    - expected length $L(C, X)$ of a symbol code C encoding an ensamble X;
    - compression issue: given an ensamble X, generate a symbol code C that...
        * is uniquely decodable,
        * is easy to decode (so, possibly, a prefix code),

    * minimizes the expected lengt $L(C, X)$;
  - Kraft inequality in the case of unique decodeability,
    * expression and proof,
    * complete symbol code,
    * given the size $|A_X|$ of an alphabet, existence of a complete, prefix code encoding it (provable by construction).

- Convex functions and Jensen's inequality:
  - convex functions and strictly convex functions;
  - Jensen's inequality,
    * proof of Jensen's inequality,
    * corollary: equality in the case of a strictly convex function.
- Kullback–Leibler divergence and Gibbs' inequality:
  - relative entropy, aka Kullback–Leibler divergence between two probability distributions;
  - Gibbs' inequality.

- Proof of Shannon's source coding theorem for symbol codes.
- A mention of...
  - Huffman *lossless* coding algorithm;
  - Lempel–Ziv *lossless* compression algorithm (LZ77);
  - (once more:) Shannon's noisy channel coding theorem.

LABORATORY CLASSES

(03/10/2018) L1. **Introduction to Verilog programming on a FPGA device. Counters and frequency dividers.**

- Safety rules.
- Introductory exercise: design of a 1 Hz counter with 8–LED array display, relying on standard analog and digital circuitry and a 8 Hz clock source.
- Basic hardware circuits:
  - frequency divider;
  - counter.

- A short overview on FPGA devices.
- Introduction to Verilog programming language:
  - module architecture;
  - example: development of a 8–bit, 1 Hz counter with an 8–LED array display.

- Basic modules implementing basic hardware circuits:
  - frequency divider;
  - counter.
- Assigned exercises:
  - development of a 8–bit, 1 Hz (or 10 Hz) counter with an 8–LED array display;
  - development of a 8–bit, 1 Hz (or 10 Hz) counter with a 2–digits BCD coding and a 2 x 4–LED array display.

(10/10/2018) L2. **Multiplexers and demultiplexers.**

- Basic hardware circuits:
  - multiplexer;
  - demultiplexer.

- Hardware and software architectures:

    - combinatorial and sequential circuits;
    - synchronous and asynchronous circuits.
- Verilog language:
    - combinatorial (*assign*) and sequential (*always*) Verilog modules.

- A basic module implementing a basic hardware circuit:
    - multiplexer.
- Assigned exercise:
    - development of a chronometer from 0 to 99.99 s (1/100 s resolution), with 2–digits BCD coding and a 2 x 4–LED array display.

(17/10/2018) L3. **Synchronous counters. Toggle flip-flops. Monostable multivibrators.**

- Risetime issue when clocking a flip-flop.

- Basic modules implementing basic hardware circuits:
    - synchronous counter with set to a preset value, and reset;
    - synchronous toggle flip-flop [*];
    - synchronous monostable multivibrator [*];
    - data latch.

  [*] to be developed within the exercises.
- Assigned exercises:
    - implementation of a synchronous module **toggle flip-flop**;
    - implementation, by means of a toggle flip-flop, of a *toggle pushbutton* to switch on/off an LED;
    - observation of the bouncing effect in a pushbutton.

(24/10/2018) L4. **Implementation of a chronometer with LCD display.**

- Blocking and non–blocking assignments.
- Basic hardware circuits:
    - peripheral device drivers.
- A basic driver module:
    - LCD driver (with display of the lap mode).
- Assigned exercise:

- final implementation of a chronometer with 1/100 s resolution, start/stop, lap/reset function, and LCD display.

(31/10/2018) L5. **Driving the DACs and the ADCs hosted on the development board. Nyquist–Shannon sampling theorem made real.**

- Solution to the exercise assigned in lab class L4:
  - chronometer.

- Numerical representation of natural and integer numbers:
  - *2's complement* representation of integers;
  - inversion of an integer $(-1) \cdot n = (\sim n) + 1$;
  - sum, difference $a - b = a + (-1) \cdot b$, multiplication $a \cdot b = |a| \cdot |b| \cdot sign(a) \cdot sign(b)$;
  - multiplication times 2^k and division by 2^k by means of the *shift* operator;

  - using the *2's complement* representation with ADC/DACs.

- Basic driver modules:
  - driver of the ADCs placed on the development board;
  - driver of the DACs placed on the development board.
- Assigned exercises:
  - evaluation of the Nyquist frequency and the transfer function gain (voltage-to-number-to-voltage) of a ADC-DAC feedthrough system;
  - implementation of a delayer.

(07/11/2018) L6. **Waveform generation.**

- Solution to the exercise assigned in lab class L5:
  - differentiator.

- A basic hardware circuit:
  - shift register.
- A basic module implementing a basic hardware circuit:
  - shift register.

- Assigned exercise:
  - implementation of sawtooth waveform generators.

(14/11/2018) L7. **Implementation of a harmonic oscillator.**

- Solution to the exercise assigned in lab class L6:
  - pseudorandom number generator.

- Theoretical and experimental aspects linked to the development of a harmonic oscillator:
  - general discussion on the difficulty of implementing an oscillator;
  - from the differential equation of a forced oscillator to the z–transform of the simulator response function $V(z)$;
  - derivation of the difference equation;
  - setting of the boundary conditions for the *cosine* operation;
  - dependency of the working frequency $f_0 = \omega_0/(2\pi)$ on the parameter k, provided that $\omega_0 T \ll 1$:
    $f_0 = f_s/(\pi 2^{\frac{k}{2}+1})$, with $f_s = 1/T$.

- Assigned exercise:
  - implementation and characterization of a harmonic oscillator.

(21/11/2018) L8. **Digital filters.**

- Solution to the exercise assigned in lab class L7:
  - implementation and characterization of a harmonic oscillator.

- Theory exercise: design of a first–order low–pass filter via bilinear transform:
  - from the Fourier transform of the real system's response function to the z–transform of the simulator's response function $V(z)$;
  - difference equation and canonical direct form II (and transposed DFII) of the simulator;
  - implementation by using a pole placed at $1 - 2^{-k}$;
  - dependency of the cutoff frequency $f_{3\,\mathrm{dB}} = (2\pi\tau)^{-1}$ on parameter k, provided that $\omega_0 T \ll 1$;
  - frequency behaviour via backward interpretation of the simulation

theorem.

- Experimental exercises:
  - implementation of a first–order low–pass filter;
  - assessment of the transfer function (to be displayed via Bode–diagrams).

(05/12/2018) A2. **Transmission lines.**
Additional lecture/lab class. Duration: 4h. Location: Electronics EduLab.

- Theory of trasmission lines:
  - useful constants,
    * $4\pi\epsilon_0 = \frac{1}{9}\,\frac{\text{nF}}{\text{m}}$, $\frac{\mu_0}{4\pi} = 10^2\,\frac{\text{nH}}{\text{m}}$,
    * $\frac{1}{\sqrt{\epsilon_0\mu_0}} = c$, $\sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0 \simeq 120\pi\,\Omega$;
  - calculation of the distributed inductance, capacitance, resistance and conductance in the case of a coaxial line (and mention of the case of a twisted pair ["doppino"]);
  - telegrapher's equations and lossless case;
  - wave equation for voltage and current in the lossless case;
  - solution of the wave equation for voltage and current in the lossless case;
  - signal speed v and characteristic impedance Z;
  - conditions of validity of the lossless case ($R' \ll \omega L'$, $G' \ll \omega C'$);
  - reflection coefficient at a load resistance R_L,
    * derivation and ...
    * case $R_L = Z$,
    * case $R_L = 0$,
    * case $R_L = \infty$;
  - reflection on case of mismatch of transmission line characteristic impedances;
  - impedance of a line section of length ℓ terminated with a resistive load R_L,
    * derivation and ...
    * case $\ell \ll \lambda$,
    * case $R_L = 0$ or $R_L = \infty$ and a mention of *stubs*,
    * case $R_L = Z$;

- a mention of half-wave dipole antenna and the value of $Z = 75\,\Omega$.
- $Z = 50\,\Omega$ as an optimal value from the constructive point of view;
- low frequency impedance matching to maximize power transmission (optimal impedance–matching between generator internal output impedance and load resistance).

- Experimental evidence of signal propagation within a 100 m long, RG58, 50 Ω coaxial cable:
  - propagation speed;
  - load resistance and reflection.

Suggested textbooks and references:

- A. V. Oppenheim, R. W. Schafer, "Digital Signal Processing", Prentice Hall;
- B. P. Lathi, "Signal Processing and Linear Systems", Oxford University Press;
- M. Hayes, "Schaums Outline of Digital Signal Processing", Schaum's Outlines.