

Department of Physics, University of Trento, a.y. 2024–2025

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 $\underline{\text{Important remark:}}$ changes on the present document are possible without any preliminary announcement.

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LECTURES

(23/09/2024) 1. Introduction to the course. Signals and systems. Sequences.

- Signal processing:
 - analog signal processing;
 - digital signal processing (DSP).
- Noise (a very short review):
 - Johnson noise;
 - shot noise.
- Signals:
 - a formal definition of signals (pictures included!);
 - continuous signals;
 - sequences (discrete-time signals).
- Sequences:
 - graphical representation;
 - remarkable sequences: impulse-sequence $\delta[n]$, step-sequence u[n];
 - delayed sequences, relation between $\delta[n]$ and u[n];
 - representation of a generic sequence by means of delayed impulsesequences;
 - periodic sequences;
 - energy of a sequence.
- Systems:
 - a formal definition of systems.

(27/09/2024) 2. LTI systems. Basic properties of LTI systems. Difference equations.

- Linear, time-invariant (LTI) systems:
 - action on a sequence;
 - impulse–response (a.k.a. transfer function) h[n], and convolution;
 - a comment on the symmetry between input x[n] and impulse-response

h[n]: their roles can be swapped!

- Properties of an LTI system:
 - reality;
 - causality;
 - (noncausality);
 - (marginal stability;)
 - bound-input, bound-output ("BIBO") stability,
 - * definition,
 - * necessary and sufficient condition for BIBO stability;
 - examples $(h[n] = \delta[n], h[n] = u[n], h[n] = a^n u[n]).$
- Difference equations:
 - similarity with the continuous case: a difference equation characterizes an LTI system and it is typically a recursive equation;
 - example,
 - * h[n] in the case of y[n] = x[n] + x[n-1] (causal and noncausal solution),
 - * Fibonacci sequence (hint at a solution via matrices; left as a homework).

(30/09/2024) 3. z-transform and its inversion.

- \bullet z-transform:
 - a discussion on the importance of changing space, in analogy with continuous systems, to solve difference equations;
 - definition and region of convergence ("ROC");
 - remarkable examples,
 - * $\delta[n]$,
 - * u[n],
 - * -u[-n-1].
- Basic properties of z-transform:
 - linearity (important: beware of the intersecting ROCs!);
 - time-shift (important: beware of new ROC!);
 - convolution theorem (important: beware of the intersecting ROCs!).
- Complementary topics: graphical representation of systems:
 - linear combinations of systems;
 - cascade systems, and invertibility of two systems (proof via z-transform).

- Summary of residue calculus:
 - $-\frac{1}{2\pi i}\oint_{\Gamma um} z_o(z-z_o)^n dz = \delta_{n,-1};$
 - $G(z) \text{ has an } n^{\text{th}} \text{-order pole in } z_o \Longrightarrow \frac{1}{2\pi i} \oint_{\Gamma \ um \ z_o} G(z) dz = \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} [G(z)(z-z_o)^n]_{z=z_o}.$
- Inversion of the z-transform:
 - derivation of the inversion expression;
 - examples,
 - * X(z) = 1 with ROC = \mathbb{C} ,

 - $* X(z) = \frac{z}{z-1} \text{ with ROC} = \{z \, | \, |z| < 1\},$ $* X(z) = \frac{z}{z-1} \text{ with ROC} = \{z \, | \, |z| > 1\}.$

(07/10/2024) 4. Basic properties of LTI systems as seen in the zdomain. Inversion of the z-transform: examples. Introduction to Nyquist-Shannon sampling theorem.

- Basic properties of systems as seen in the z-domain:
 - reality.
 - * reality $\iff \overline{X}(\overline{z}) = X(z),$
 - * examples $(\delta[n], a^n u[n], -a^n u[-n-1]),$
 - * existence of complex-conjugate zeroes and poles;
 - causality,
 - * beware: causal $\neq \overline{\text{noncausal}} (k \cdot \delta[n] \dots \text{ is both!}),$
 - * causality, namely x[n] = 0 for $n < 0 \iff \infty \subset ROC$ (implication \Leftarrow via Taylor-Laurent series),
 - * noncausality, namely x[n] = 0 for $n > 0 \iff 0 \in ROC$ $(implication \Leftarrow via Taylor-Laurent series),$
 - * causuality & noncausality, namely x[n] = 0 for $n \neq 0$, i.e. $x[n] = k \cdot \delta[n]$ with $k \in \mathbb{C} \iff$ $0 \in ROC, \infty \subset ROC \iff X(z)$ uniform on \mathbb{C}

(the statement corresponds to the Liouville theorem: a bounded holomorphic function whose ROC coincides with \mathbb{C} is uniform),

- * examples $(\delta[n], a^n u[n], -a^n u[-n-1]);$
- BIBO stability,
 - * BIBO stability $\iff \Gamma_1 \subset ROC$,
 - * examples $(\delta[n], a^n u[n], -a^n u[-n-1]).$
- Remarkable examples concerning the inversion of the z-transform:
 - Fibonacci sequence;

- Poisson process and distribution.
- Towards Nyquist-Shannon sampling theorem:
 - sampling.
 - * generation of a sequence x[n] starting from a continuous signal $x_a(t)$: $x[n] = x_a(nT),$
 - * basic issue: how much information is lost in the sampling process, or, better, how good a continuous signal can be reconstructed from a sequence?
 - important definitions,
 - * sampling time/period T,
 - * sampling frequency/rate $f_s = \frac{1}{T}$,
 - * sampling frequency $\omega_s = \frac{2\pi}{T}$,
 - * Nyquist frequency $f_{\text{Ny}} = \frac{1}{2T} = \frac{f_s}{2}$, $\omega_{\text{Ny}} = \frac{\pi}{T}$,
 - * Nyquist band $\left(-\frac{\pi}{T}, \frac{\pi}{T}\right)$.

(11/10/2024) 5. Nyquist-Shannon sampling theorem.

- Nyquist–Shannon precursor relation, reconstruction:
 - starting conditions:
 - 1a) analog, continuous signal $x_a(t) \in L^2$ and
 - 1b) related sampled sequence x[n] being BIBO stable;
 - "Nyquist-Shannon precursor relation" between the \mathcal{F} -transform $\tilde{X}(\omega)$ of $x_a(t)$ and the z-transform X(z) of x[n]: $T \cdot \tilde{X}(e^{-i\omega T}) = \sum_{\forall k} \tilde{X}(\omega + \frac{2\pi}{T}k)$ (where both terms are periodic);
 - useful definitions I. (ex post), II., III. (ex ante),
 - I. "folding", $\tilde{X}_{folding}(\omega) \equiv \sum_{\forall k} \tilde{X}(\omega + \frac{2\pi}{T}k)$,
 - II. $\frac{\pi}{T}$ -window, $W(\omega) \equiv [\theta(\omega + \frac{\pi}{T}) \theta(\omega \frac{\pi}{T})],$
 - III. "reconstruction", $x_{rec}(t) \equiv \sum_{\forall k} x[k] \operatorname{sinc} \left[(t kT) \frac{\pi}{T} \right];$
 - reconstruction,
 - * $\mathcal{F}^{-1}\left[W(\omega) \cdot \tilde{X}_{folding}(\omega)\right] = x_{rec}(t),$ * remarkably, $x_{rec}[n] = x_a(nT) = x[n].$
- Nyquist-Shannon sampling theorem:
 - final condition (the crucial one!):
 - 2) the analog, continuous signal $x_a(t)$ is $\frac{\pi}{T}$ -BL (band-limited);

- Nyquist–Shannon sampling theorem in the frequency domain), $\tilde{X}(\omega) = W(\omega) \cdot \tilde{X}_{folding}(\omega)$;
- Nyquist–Shannon sampling theorem in the time domain, $x_a(t) = x_{rec}(t)$.
- Aliasing in the case of a non- $\frac{\pi}{T}$ -BL signal $x_a(t)$:
 - Aliasing: if condition 2) is not met, i.e. $x_a(t)$ is not $\frac{\pi}{T}$ -BL, then $\tilde{X}(\omega) \neq W(\omega) \cdot \tilde{X}_{folding}(\omega)$, and $x_{rec}(t) \neq x_a(t)$, i.e. $x_{rec}(t)$ is not equal to $x_a(t)$ but it is something else (\sim latin alias).
- Practical statement of Nyquist–Shannon sampling theorem: if f_0 is the maximum frequency occurring in a signal, use a sampling frequency f_s such that $f_s > 2f_0$.
- Remarkable example: aliasing in the case of sinusoidal and cosinusoidal signals.

(14/10/2024) 6. Digital simulation of an analog system.

- Theorem concerning the digital simulation of an analog system ("simulation theorem"):
 - general discussion;
 - proof assuming...
 - * 1a) Fourier-transformable input signals and transfer function, i.e. $x(t), G(t) \in L^2$,
 - * 1b) BIBO-stable simulator h[n],
 - * 2) $\frac{\pi}{T}$ -BL input signals;
 - expression of the ideal transfer function h[n] of the simulator;
 - reality: G(t) is real $\Rightarrow h[n]$ is real.
- Implementation issues:
 - stability of the ideal transfer function h[n] is to be assessed case by case;
 - non-causality issue for the ideal transfer function h[n] (example: first-order low-pass filter);
 - difficulty in the general case of calculating h[n] (example: first-order low-pass filter);
 - a remarkable example: digital differentiator (ideal solution $\propto (1 -$

$$\delta[n])\frac{(-1)^n}{n}$$
).

- "Backward interpretation of the simulation theorem":
 - statement: any digital system characterized by an impulse sequence v[n]—and the related z-transform V(z)—is a perfect simulator of an analog system whose impulse response's Fourier transform $\tilde{V}(\omega)$ is given by $V(e^{-i\omega T})$.
- Approximated simulation via "backward interpretation of the simulation theorem":
 - an approach to overcome the implementation issues: requiring the simulator's output y'[n] to approximate the sampled analog output $y[n], y'[n] \cong y[n]$, rather than imposing h[n] = g[n].
 - So, because it is mostly impossible to find H(z), and thus h[n], such that it exactly simulates a given $\tilde{G}(\omega)$,
 - find an implementable V(z), and thus v[n], such that its Fourier-transform equivalent function $V(e^{-i\omega T})$ suitably approximates $\tilde{G}(\omega)$;
 - (approximation of an ideal system h[n], H(z) through a real one v[n], V(z) via, ex. gr., minimization of the Tchebycheff error or the root–mean–square error;)
 - a remarkable example: digital differentiator,
 - * solution $\delta[n] \delta[n-1]$,
 - $\ast\,$ solution via a non recursive (FIR) filter based on delays up to 2 periods.

(21/10/2024) 7. Simulation via bilinear transform.

- Bilinear transform:
 - an issue: how to express ω , or s, in terms of z by relying on $z = e^{-i\omega T}$ and exploiting the approximation approach to simulation provided by the backward interpretation of the simulation theorem;
 - $\begin{array}{l} \text{ bilinear transform statement } \omega \to \frac{2i}{T}\frac{z-1}{z+1}, \ s \to \frac{2}{T}\frac{z-1}{z+1} \\ \Longrightarrow V(z) = \tilde{H}\left(\omega = \frac{2i}{T}\frac{z-1}{z+1}\right); \ V(z) = \tilde{H}_{\text{Laplace}}\left(s = \frac{2}{T}\frac{z-1}{z+1}\right); \end{array}$
 - desirable frequency behaviour for $T \ll bandOfInterest^{-1}$.
- Simulation via bilinear transform of an LTI system characterized by a rational transfer function:
 - basic properties,
 - * reality $(h(t) \text{ is real } \iff \tilde{H}^*(\omega) = \tilde{H}(-\omega)),$
 - * causality (to be imposed),
 - * BIBO-stability, h(t) is BIBO-stable \iff stable system with rational transfer function $\tilde{H}(s)$,

$$\begin{array}{ccc} \cdot & s \to \frac{2}{T} \frac{z-1}{z+1} & \Longleftrightarrow & z \to \frac{sT/2+1}{sT/2-1}, \\ \cdot & \Longrightarrow & \operatorname{Re} s < 0 \leftrightarrow |z| < 1; \end{array}$$

- in the case of systems characterized by rational transfer functions $\tilde{H}(\omega) = N(\omega)/D(\omega)$, with degree $[N(\omega)] < \text{degree}[D(\omega)]$, superiority of bilinear transform with respect to other transforms (for example the one in which $\tan(\omega T/2)$ is replaced by $\sin(\omega T)$), due to a desirable behaviour in the neighbourhood of the Nyquist frequency.
- Example:
 - low-pass filter simulator via bilinear transform;
 - implementation via a difference equation;
 - frequency response.

(21/10/2024) 7. Simulation via bilinear transform.

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- Example:

- first-order low-pass filter simulator via bilinear transform;
- implementation via a difference equation;
- frequency response.

(28/10/2024) 8 Filter design.

- A recap on designing a first-order low-pass filter via bilinear transform:
 - derivation of V(z) via bilinear transform;
 - basic properties,
 - * reality,
 - * causality (to be imposed),
 - * BIBO-stability;
 - desirable behaviour in the neighbourhood of the Nyquist frequency;
 - implementation via a difference equation;
 - transfer function $\tilde{V}(\omega)$ via "backward interpretation of the simulation theorem".
- ullet Structure of a filter implementing a rational function in the z-domain:
 - effect of zeroes and poles on $\tilde{V}(\omega)$;
 - example: design of a bandpass filter at $f = f_{Nyquist}/4 = f_{sampling}/8$:
 - * positioning of zeroes and poles, by taking into account reality, causality, BIBO–stability,
 - * final expression for V(z),
 - * a discussion on the frequency response $\tilde{V}(\omega)$,
 - * inversion of z-transform V(z) and practical implementation;
 - example: design of a notch filter at $f = f_{Nyquist}/2 = f_{sampling}/4$:
 - * positioning of zeroes and poles, by taking into account reality, causality, BIBO-stability,
 - * final expression for V(z),
 - * frequency response $\tilde{V}(\omega)$,
 - st inversion of z-transform V(z) and practical implementation.

(04/11/2024) 9. Discrete-Fourier-Transform (DFT).

- Discrete–Fourier–Transform (DFT):
 - assumption of the periodicity of $x_a(t)$, with period NT, $N \in \mathbb{N}^+$ and calculation of the coefficients of the Fourier series;
 - assumption of the $\frac{\pi}{T}$ -band-limitedness of $x_a(t)$;

- (assumption of N being even);
- derivation of DFT and relation with the Fourier series coefficients;
- inversion, periodicity, Parseval's theorem.
- Matrix representation of DFT:
 - notation,
 - * $f_n \equiv f[n],$
 - * twiddle factor $W_N \equiv e^{2\pi i/N}$;
 - matrix representation;
 - dependence $O(N^2)$ of the number of operations required for the DFT on the dimension N of the input string.
- Example: DFT of $\cos(2\pi ft)$ with $f = 2 \cdot \frac{1}{NT}$ in the case N = 8.
- A short mention on DFT (and FFT) of a nonperiodic function and windowing.

(11/11/2024) 10. Fast-Fourier-Transform (FFT).

- A summary of last lecture.
- Radix-2 decimation-in-time Fast-Fourier-Transform (FFT) algorithm:
 - short history: Gauss 1805, Cooley and Tukey 1965;
 - Danielson-Lanczos lemma (decimation, i.e. split);
 - butterfly diagram (basic computational unit);
 - element ordering via bit swapping.
- Dependence $O(N \log N)$ of the number of operations required for the FFT on the dimension N of the input string.
- Examples of FFT in the case N = 8:
 - FFT of $\cos(2\pi ft)$ with $f = 2 \cdot \frac{1}{NT}$;
 - generic sequence $\{f_0, f_1, \ldots, f_7\}$, and equivalence with the DFT;
 - triangular wave $\{0, 1, 2, 3, 4, 3, 2, 1\}$;
 - ... proposed as a homework,
 - * FFT of $\sin(2\pi ft)$ with $f = 2 \cdot \frac{1}{NT}$,
 - * FFT of a constant,
 - * FFT of $\delta[n]$,
 - * FFT of $\delta[n-3]$,
 - * FFT of $\delta[n-4]$.

(18/11/2024) 11. An introduction to information theory: Shannon's source coding theorem for symbol codes.

- The basic issue: a definition of information, i.e., how it can be measured:
 - Shannon's approach (1948) with uncertainty;
 - example of a sport match.
- Convex functions and Jensen's inequality:
 - convex functions and strictly convex functions;
 - Jensen's inequality,
 - * proof of Jensen's inequality,
 - \ast corollary: equality in the case of a strictly convex function.
- Kullback–Leibler divergence and Gibbs' inequality:
 - relative entropy, aka Kullback–Leibler divergence between two probability distributions;
 - Gibbs' inequality.
- Ensambles (Khinchin's finite schemes; "Experiment") as triplets made of...:
 - random variable;
 - alphabet of symbols (or vocabulary of words), each corresponding to a realization of the r.v.;
 - probability distribution of the r.v.
- Binary symbol codes and compression issue:
 - definitions related to symbol codes,
 - * (binary) symbol code, codewords, length of codewords,
 - * uniquely decodable symbol code and prefix code, a prefix code is uniquely decodable; the contrary is not true (counterexample {1, 101}),
 - a prefix code is (generally) easy to decode,
 - * examples;
 - expected length L(C, X) of a symbol code C that encodes an ensamble X;
 - compression issue: given an ensamble X, generation of a symbol code C that...
 - * (compulsory) is uniquely decodable,
 - * (desirable) is easy to decode (so, possibly, a prefix code),
 - * (compulsory) minimizes the expected length L(C, X);

- Kraft inequality in the case of unique decodeability,
 - * expression and proof,
 - * definition of a complete symbol code,
- Shannon's source coding theorem for symbol codes.

(25/11/2024) 12. An introduction to information theory: Shannon entropy of an ensamble; compression.

- A summary of last lecture on Shannon's source coding theorem:
 - (uniquely decodable, and possibly prefix) binary symbol codes and compression;
 - expected length $\ell(C,X)$ of a symbol code C encoding an ensamble X;
 - compression issue: given an ensamble X, generate a symbol code C that...
 - * is uniquely decodable, possibly a prefix one, and
 - * minimizes the expected length $\ell(C, X)$;
 - statement of Shannon's source coding theorem (part I): $\ell(C, X) \geqslant H(X)$.
- Shannon entropy of an ensamble, and a mention of uniqueness theorem and of noisy channel coding theorem:
 - from "information" to "entropy" (a uniform distribution maximizes H(X)...);
 - a mention of,
 - * Shannon's source coding theorem, part II, namely a uniquely decodable code exists such that $\ell(C, X) < H(X) + 1$,
 - * Shannon-Khinchin axioms and uniqueness theorem,
 - * noisy channel coding theorem.
- \bullet Huffman lossless coding algorithm:
 - algorithm;
 - example;
 - properties,
 - * Huffman algorithm generates prefix symbol codes (provable by construction),
 - * Huffman is complete (provable by construction),
 - * Huffman coding is optimal (statement only),
 - * (a drawback:) necessity of knowing p(x) in advance;

- compression improvement via "syllables", and example with p(a)=1/4, p(b)=3/4.
- \bullet Lempel–Ziv lossless compression algorithm (LZ77):
 - algorithm;
 - $-\ {\rm example}.$
- A mention of lossy compression algorithms.

LABORATORY CLASSES

(04/10/2024) L0. Software setup session. (2 hours)

- Setting up a Linux environment.
- Walkthrough to set up Vivado.
- Compilation and device programming tests.

(08/10/2024) L1. Introduction to Verilog programming on a FPGA device. Counters and frequency dividers.

- A short overview on FPGA devices and on HDL programming languages.
- Introductory problem: design of a 1 Hz counter with 8–LED array display, relying on standard analog and digital circuitry and an 8 Hz clock source.
- Basic hardware circuits:
 - frequency divider;
 - counter.
- Verilog programming language:
 - module architecture;
 - template example: development of an 8-bit, 1 Hz counter with an 8-LED array display, working from 0 to 255;
 - basic modules implementing basic hardware circuits:
 - * frequency divider;
 - * counter.

• Problems:

- development of an 8-bit, 1 Hz counter with an 8-LED array display, working from 0 to 9;
- development of an 8–bit, 10 Hz counter with an 8–LED array display, working from 0 to 9.
- development of a 10 Hz counter, working from 0 to 99, with a 2-digits
 BCD coding and a 2 x 4-LED array display;
- Additional problems:
 - development of a 10 Hz down counter.

(09/10/2024) L2. Multiplexers and demultiplexers. Synchronous counters.

- Solutions to the problems assigned in the previous lab class.
- Hardware and software architectures:
 - combinatorial (are made of gates, and depend on states) and sequential circuits (are made of flip-flops [and gates], and depend on clocks [and states]);
 - synchronous and asynchronous circuits;
 - example: asynchronous counter and synchronous version by means of a finite-state machine.
- Basic hardware circuits:
 - multiplexer;
 - demultiplexer.
- Verilog programming language:
 - combinatorial (assign) and sequential (always) Verilog modules;
- Problems:
 - development of a stop watch from 0 to 99.99 s (1/100 s resolution), with 2–digits BCD coding and a 2 x 4–LED array display;
 - development of a syncronous counter based on a "master clock".

(10/10/2024) L3. Toggle flip-flops. Monostable multivibrators.

- Solutions to the problems assigned in the previous lab class.
- Risetime issue when clocking a flip-flop.
- Verilog programming language:
 - basic modules implementing basic hardware circuits:
 - * multiplexer (synchronous and asynchronous version);
 - * synchronous counter with set to a preset value, and reset.
 - basic modules implementing basic hardware circuits:
 - * synchronous toggle flip-flop [*];
 - * synchronous monostable multivibrator [*].
 - [*] to be developed within the problems.

• Problems:

- implementation of a synchronous module toggle flip-flop and, through this, of a push light switch to switch on/off an LED; observation of the bouncing effect in a pushbutton;
- implementation of a synchronous module **monostable multivibrator** and, through this, of a timer to switch on an LED for a 1 second;
- implementation, by means of a monostable multivibrator and a toggle flip-flop, of an improved *push light switch* to switch on/off an LED; solution to the bouncing effect in a pushbutton.

(18/10/2024) L4. Finite-state machines.

- Solutions to the problems assigned in the previous lab class.
- Finite-state machines.
- Basic hardware circuits:
 - data latches;
 - shift registers.
- Verilog programming language:
 - peripheral device drivers (ex.gr.: OLED driver with display of the lap mode).

• Problems:

- implementation of a synchronous 5-state finite-state machine with two pulse control;
- final implementation of a stopwatch with 1/100 s resolution, start/stop, lap/reset function, and OLED display.

(23/10/2024) L5. Nyquist–Shannon sampling theorem made real. Waveform generation.

- Using the 2's complement representation with ADC/DACs.
- Verilog programming language:
 - driver of the ADCs placed on the development board;
 - driver of the DACs placed on the development board.
- Problems:

- evaluation of the Nyquist frequency and the transfer function gain (voltage-to-number-to-voltage) of a ADC-DAC feedthrough system;
- implementation of a delayer;
- implementation of sawtooth waveform generators.

(30/10/2024) L6. Implementation of a harmonic oscillator.

- Theoretical and experimental aspects linked to the development of a harmonic oscillator:
 - general discussion on the difficulty of implementing an oscillator;
 - from the differential equation of a forced oscillator to the z-transform of the simulator response function V(z);
 - derivation of the difference equation;
 - setting of the boundary conditions for the *cosine* operation;
 - dependency of the working frequency $f_0 = \omega_0/(2\pi)$ on the parameter k, provided that $\omega_0 T \ll 1$: $f_0 = f_s/(\pi 2^{\frac{k}{2}+1})$, with $f_s = 1/T$.
- Practical demonstration of FFT windowing.
- Problems:
 - implementation and characterization of a harmonic oscillator.

(06/11/2024) L7. Digital simulation of analog filters.

- A summary on Bode diagrams (and on frequency roll-off in filters).
- Theory problem: design of a first-order low-pass filter via bilinear transform:
 - from the Fourier transform of the real system's response function to the z-transform of the simulator's response function V(z);
 - difference equation and block diagram of the simulator;
 - implementation by using a pole placed at $1-2^{-k}$;
 - dependency of the cutoff frequency $f_{3 \text{ dB}} = (2\pi\tau)^{-1}$ on the parameter k, provided that $\omega_0 T \ll 1$;
 - frequency behaviour via "backward interpretation of the simulation theorem".
- Problems:

- implementation of a first-order low-pass filter;
- experimental check of the theoretical dependence of the cutoff frequency on the parameter k;
- $-\,$ assessment of the transfer function (to be displayed via Bode–diagrams) for a specific cutoff frequency.

(13/11/2024) L8. Simulation of a final exam.

 $\bullet\,$ Simulation of a final exam.